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EFFECT OF THERMAL PROPERTIES OF BOUNDARIES ON STABILITY OF CONVECTIVE FLOW IN A VERTICAL LAYER

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UDC 532.516

The results of a solution of the problem of the stability of steady convective flow in a vertical layer with thermally insulated boundaries and a comparison with the opposite limiting case of ideally thermally conducting boundaries are presented.

The results of studies of the stability of closed steady convective flow between vertical parallel planes [1,2] show that depending on the value of the Prandtl number Pr the instability is caused by mechanisms which differ in their physical nature. At low and moderate Prandtl numbers hydrodynamic disturbances leading to the formation of steady vortices at the interface of the opposing flows are responsible for the instability. At larger Prandtl numbers ($Pr > 12$) the instability has a wave nature and is connected with an increase in the convective fluxes of temperature waves.

The numerical results presented in [1,2] were obtained on the assumption that temperature disturbances vanish at the boundaries of the layer. Such boundary conditions correspond to the limiting case when the thermal conductivity of the boundaries is much greater than the thermal conductivity of the liquid. If the thermal conductivities of the liquid and the solid masses bordering on it are comparable then temperature disturbances penetrate into the solid masses. Then the question arises of whether the relative thermal conductivity of the boundaries affects the stability of the convective flow (the conjugate problem of stability of convective flow). It is clear in advance that the hydrodynamic mechanism of the instability must be little sensitive to the thermal properties of the solid masses. As for a wave instability, since it is connected with growing temperature waves it could be expected, generally speaking, that the properties of the solid masses have a considerable effect on the critical parameters of this instability. The results presented below show, however, that the penetration of temperature disturbances into the surrounding solid masses has a weak effect on the conditions of formation of instabilities of both the hydrodynamic and the wave types.

To clarify the role of the penetration of thermal disturbances on the stability it is obviously sufficient to consider the limiting case opposite to that which one usually has in mind, namely, when the thermal conductivity of the liquid is far larger than the thermal conductivity of the boundaries. In this limiting case the boundary condition of thermal insulation must be set up for temperature disturbances.

In the closed vertical layer between the planes $x = \pm h$ a plane-parallel convective flow is established with a linear temperature profile and a cubic velocity profile:

Perm' State University. Perm' Pedagogical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 6, pp. 1062-1064, June, 1977. Original article submitted May 4, 1976.

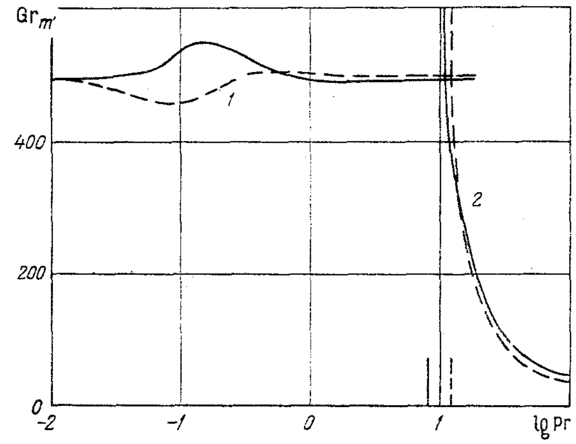

Fig. 1. Minimum critical Grashof number as a function of Prandtl number: 1) hydrodynamic branch; 2) wave branch. Solid lines: thermally insulated boundaries; dashed lines: ideally thermally conducting boundaries.

$$T_0 = -x, \quad v_0 = \frac{\mathrm{Gr}}{6}(x^3 - x). \tag{1}$$

Here we introduce dimensionless variables; as the units of distance, velocity, and temperature we take h, ν/h (ν is the kinematic viscosity), and the half-difference Θ between the temperatures of the planes. The Grashof number is defined as usual: $\mathrm{Gr} = g\beta\Theta h^3/\nu^2$. The analysis of small normal disturbances of the form $\exp(-\lambda t + ikz)$ leads to dimensioness equations for the amplitudes of the disturbances of the stream function φ and the temperature θ:

$$(\varphi^{IV} - 2k^2\varphi'' + k^4\varphi) + (\varphi'' - k^2\varphi)(\lambda - ikv_0) + ikv_0''\varphi - \mathrm{Gr}\,\theta' = 0, \tag{2}$$

$$\frac{1}{\mathrm{Pr}}(\theta'' - k^2\theta) + (\lambda - ikv_0)\theta - ikT_0'\varphi = 0 \tag{3}$$

(here $\mathrm{Pr} = \nu/\chi$ is the Prandtl number). The following conditions for the amplitudes are set up at the boundaries of the layer:

$$x = \pm 1: \quad \varphi = \varphi' = \theta' = 0. \tag{4}$$

The boundary problem (2)-(4) determines the spectrum of the characteristic disturbances and their decrements λ. The limits of stability are found from the condition $\lambda_r = 0$. The solution of the problem was found numerically by the Runge — Kutta — Merson method with orthogonalization of the vector solutions by the Gram — Schmidt method at each step of integration; the orthonormalization was performed with respect to the maximum vector solution in absolute value (in the given step).

The principal result of the calculations is presented in Fig. 1, where the dependence of the minimum (with respect to k) critical Grashof number $\mathrm{Gr_m}$ on the Prandtl number Pr is shown for the hydrodynamic (1) and wave (2) branches of instability. The corresponding limits of stability for ideally conducting boundaries [1, 2] are shown here by a dashed line for comparison. As is seen, in both branches of instability the dependences $\mathrm{Gr_m}(\mathrm{Pr})$ for the two types of boundary conditions are similar. By comparison with the case of ideally conducting boundaries there is some decrease in the limiting Prandtl number Pr_* beginning with which the wave branch of instability appears (extrapolation gives a value of $\mathrm{Pr}_* \approx 8$ instead of 11.4 for the case of ideally thermally conducting boundaries). In the limit of $\mathrm{Pr} \gg \mathrm{Pr}_*$, as an asymptotic analysis shows (see [3]), the same limiting law $\mathrm{Gr_m} = 590/\sqrt{\mathrm{Pr}}$ occurs in both cases of boundary conditions. The critical values of the wave number k_m are also similar for the two variants of the boundary conditions discussed.

Thus, the calculation shows that the thermal properties of the boundaries have a weak effect on the stability of convective flow in a vertical layer. In this sense one must emphasize the difference from the problem of the stability of equilibrium of a horizontal layer of liquid heated from below, where, as is known (see [1]), there is a very strong dependence of the limit of stability and the form of the disturbances on the thermal properties of the boundary solid masses.

NOTATION

x, z, transverse and longitudinal coordinates; v_0, T_0, undisturbed velocity and temperature profiles; φ, θ, amplitudes of disturbances of stream function and temperature; λ, decrement; k, wave number.

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HYDRODYNAMIC STABILITY OF CONVECTIVE FLOW OF A NON-NEWTONIAN FLUID IN A VERTICAL LAYER

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UDC 536.25:532.135

Steady convective non-Newtonian fluid flow and its stability under small perturbations are investigated.

We wish to analyze the free thermal convection of a non-Newtonian fluid in an infinite-plane vertical channel. We use the rheological equation

$$\tau_{ij} = -\delta_{ij}p + \eta(1 + aI)^{n-1}e_{ij}. \tag{1}$$

Transition to a Newtonian fluid takes place as $a \to 0$ or $n \to 1$. In the limit of large a the Ostwald–Deville model is obtained from (1). Unlike the power-law model, Eq. (1) gives a finite initial viscosity.

It has been shown [1] that Eq. (1) well describes the rheological properties of polymer solutions in a definite concentration interval. The authors of [1] discuss pseudoplastic media with $n - 1 = -m < 0$.

We now investigate plane convective motion homogeneous along the z axis. We place the coordinate axes so that the y axis is directed upward along the centerline of the channel and the x axis is perpendicular to the walls. The wall coordinates are $x = \pm h$. The walls are maintained at constant temperatures: $T(-h) = \Theta_0$; $T(h) = -\Theta_0$.

We adopt the following reference units: distance h; time $h^2\rho/\eta$; velocity $\rho g\beta\Theta_0 h^2/\eta$; temperature Θ_0; pressure $\rho g\beta\Theta_0 h$. The system of dimensionless free-convection equations in projections onto the x and y axes has the form

$$\frac{\partial v_x}{\partial t} + \mathrm{Gr}\left(v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left[H\Delta v_x + 2\frac{\partial H}{\partial x}\cdot\frac{\partial v_x}{\partial x} + \frac{\partial H}{\partial y}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right]; \tag{2}$$

$$\frac{\partial v_y}{\partial t} + \mathrm{Gr}\left(v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y}\right) = -\frac{\partial p}{\partial y} + \left[H\Delta v_y + \frac{\partial H}{\partial x}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) + 2\frac{\partial H}{\partial y}\cdot\frac{\partial v_y}{\partial y}\right] + T; \tag{3}$$

$$\frac{\partial T}{\partial t} + \mathrm{Gr}\,\vec{v}\nabla T = \mathrm{Pr}^{-1}\Delta T; \tag{4}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0; \tag{5}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 6, pp. 1065-1070, June, 1977. Original article submitted May 4, 1976.